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# A ccuracy and Preferences for Legal Error

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### Accuracy and Preferences for Legal Error

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#### Abstract

We study how legal procedures may evolve over time in response to technological advancements which increase the accuracy of evidence collection methods. First, we show that accuracy and type-1 errors (wrongful findings of liability) must reduce each other's effectiveness in mitigating optimal type-2 errors (wrongful failures to assign liability) for previous results in the literature to hold. When this condition holds, for major crimes the median voter's tolerance for type-1 errors is reduced as the legal system's accuracy increases. However, this relationship need not hold for minor offenses. Our analysis also reveals that legal processes that emerge under electoral pressures convict more often than is optimal but less often than necessary to maximize deterrence. Moreover, when the median voter's preferences are implemented, an increase in accuracy can counterintuitively reduce welfare.

**Keywords:** Crime, deterrence, legal errors, accuracy, standard of proof, election.

JEL classification: K4.

#### 1 Introduction

How do people's preferences over legal error change as technological advances make more conclusive evidence collection possible and thereby enhance accuracy? The history of many legal institutions, which have evolved from employing high type-1 error procedures (wrongful findings of liability) that came close to employing a presumption of guilt to requiring proof beyond a reasonable doubt for all elements of a crime in the United States (*In Re Winship* 1970), suggests that enhanced accuracy may tilt people's preferences towards lower type-1 errors.<sup>1</sup> Moreover, a simple casual observation suggests that there is

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<sup>&</sup>lt;sup>1</sup>See, e.g., Smith (1950), arguing that even as recently as in the early nineteenth century "many English criminal defendants ... did not benefit from a presumption of innocence but, rather, struggled against a statutory presumption of guilt."

correlation in the direction that supports this hypothesis between a country's technological development and its citizens' preferences over type-1 versus type-2 errors (wrongful failures to find liability). This intuitive relationship between accuracy and people's relative preference for type-1 errors is illustrated in figure 1.<sup>2</sup> The horizontal axis measures the ICT Development Index (IDI), which is a composite index designed to measure countries' information and communication technologies. On the vertical axis, we have the fraction of people (among those who have reported a strict preference) who believe type I errors are worse than type II errors. These preferences are taken from the 2006 International Social Survey Program Role of Government survey. The survey includes 35 countries.<sup>3</sup>

<sup>&</sup>lt;sup>2</sup>Of course, many additional factors can contribute to cross-country differences as well changes over time in people's preferences over legal errors (see, e.g. Givati 2019 and Johnson and Koyama 2014). We present figure 1 only to highlight an intuitive relationship which motivates our research question, rather than claiming that it alone demonstrates any kind of causal relationship.

<sup>&</sup>lt;sup>3</sup>Note that Taiwan does not appear on the scatter diagram since we have no IDI index for this country.

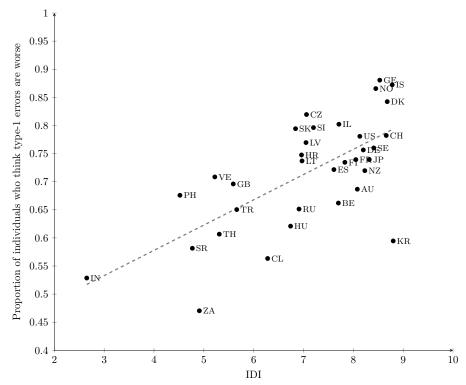


Figure 1: ICT Development Index and relative preference for type-1 errors

Sources: IDI Index: Measuring the Information Society - Report 2017. Opinions on legal errors are taken from the 2006 International Social Survey Program Role of Government survey.

In this article, we present a simple law enforcement model to investigate the relationship between increased accuracy and the composition of optimal as well as median voters' most preferred legal errors. We note that legal systems are imperfect, and they necessarily produce both type-1 and type-2 errors. At least two factors that can impact these errors have been analyzed in the literature. First, the accuracy of the legal system, which pertains to the informativeness of legal evidence, can presumably reduce both types of errors. Second, given any level of accuracy of the legal system, governments can impact the distribution of type-1 and type-2 errors by changing procedural rules, including the standard of proof applicable in various legal proceedings.<sup>4</sup>

We study the interplay between these two sources of variation in judicial errors by questioning how legal procedures would likely respond to changes in the

<sup>&</sup>lt;sup>4</sup>In the economics literature, various standards of proof (including preponderance of the evidence, and proof beyond a reasonable doubt, which are the most frequently used standards in the civil and criminal settings, respectively) are often conceived of as threshold levels of evidence which must be met for a finding of liability. See, e.g., the references in note 5, below.

accuracy of the legal system. To answer this question, we start by identifying the technical properties that legal procedures must possess for increases in accuracy to result in changes in the optimal legal procedures which are consistent with intuitive assumptions employed in the prior literature. In particular, Kaplow and Shavell (1994) assume that increases in accuracy result in reductions in both type-1 and type-2 errors. Contrary to this assumption, we show that when the state can adjust its legal procedures in response to changes in accuracy, it is possible that it may do so in a way that increases one type of error while reducing the other. We then show that for this possibility to be ruled out, marginal reductions in type-2 errors obtained by marginal increases in type-1 errors must be decreasing in the accuracy of the legal system around the deterrence maximizing policy. In other words, accuracy and type-1 errors must reduce each other's effectiveness in reducing type-2 errors.

Next, assuming this technical property holds, we question how citizens' preferences for legal procedures may respond to technological advancements which increase the accuracy of legal proceedings. Consistently with these observations, our model suggests that the median voter's preferred type-1 error is decreasing in accuracy under a broad range of circumstances. However, quite importantly, although this relationship is likely to hold true for serious crimes which result in large harms, they need not hold for minor infractions where the harm is relatively small. Thus, our model supplies a rationale for the intuitive relationship between accuracy and people's most preferred legal procedures pertaining to serious crimes, while also explaining why, even modern and well developed nations may not place much of a value on mitigating type-1 errors pertaining to minor infractions.

Although the primary focus of our article is the relationship between accuracy and legal standards, in conducting our analysis we uncover results pertaining to closely related issues as well as the prior literature. In particular, we show that the median voter's most preferred legal procedures lie in between the welfare maximizing and deterrence maximizing procedures. Moreover, we show that an increase in accuracy can counter-inuitively reduce welfare when the median voter's preferences are implemented. These results relate to the political economics of law enforcement literature, which questions how public enforcers who face election pressures may deviate from optimal policies (Friehe and Mungan 2020, Langlais and Obidzinski 2007, Mungan 2017, Obidzinski 2019), as well as the extensive literature identifying deterrence and welfare maximizing legal standards.<sup>5</sup> Our results add to the first line of research by showing that if legal systems are shaped by electoral pressures, they are likely to result in over-punitive institutions, as in the prior literature. They complement the second line of research by showing that, although welfare maximizing legal procedures may require sacrificing a large degree of deterrence, standards

<sup>&</sup>lt;sup>5</sup>This literature is quite extensive and has been expanding rapidly recently. See, e.g., Demougin and Fluet 2005, 2006, Fluet and Mungan 2020, Garoupa 2018, Kaplow 2011, 2012, Kaplow and Shavell 1994, Lando 2009, Lando and Mungan 2018, Miceli 1990, Mungan 2011, 2020a, 2020b, Mungan and Samuel 2019, Mungan and Wright 2019, Obidzinski and Oytana 2019, 2020, Rizzolli and Saraceno 2013, Yilankaya 2002.

that emerge in systems prone to electoral pressures will not do so. Our primary focus relates these extensive two strands of the literature to the less studied issue of accuracy (Kaplow and Shavell 1994, Kaplow 2012, and Obidzinski 2019). In doing so, in addition to uncovering implicit technical assumptions made in the prior literature, we provide a positive explanation for the intuitive —and arguably observed—relationship between accuracy and citizens' preferred legal standards. Additionally, we show that the presence of political pressures can cause increased accuracy to reduce rather than enhance welfare as is assumed in prior work analyzing accuracy.

To formalize the results we describe above, in the next section we start by deriving conviction probabilities as a function of legal standards employed and accuracy. Then, in section 3, we study how accuracy affects the median voter's most preferred legal procedures. In section 4, we discuss the implications of our findings and conclude.

#### 2 Conviction Probabilities

We consider a simple enforcement model wherein potential offenders may commit crime. The payoffs associated with committing a crime and refraining from doing so are naturally affected by the probabilities of conviction that are attached to each option. Because detection of crime is imperfect, a person who does not commit crime may nevertheless be convicted with probability  $\alpha$ , and a person who commits crime may nevertheless escape conviction with probability  $1-\beta$ . Since these probabilities are central to our analysis, before describing the remaining components of the model, we first explain the relationship between  $\beta$  and  $\alpha$  in detail. In what follows, we often refer to  $\alpha$  simply as 'type-1 error' as opposed to 'the probability of type-1 error' to abbreviate descriptions and derivations. We follow the same approach for  $1-\beta$ .

#### 2.1 Probabilities of Conviction

The probabilities of conviction,  $\beta$  and  $\alpha$ , refer to the overall conviction probabilities associated with the legal system (as opposed to conviction probabilities conditional on adjudication, for instance), because our objective is to identify a relationship between the overall accuracy of the legal system and citizens' tolerance levels towards wrongful convictions. We assume that various legal procedures in the legal system, including the standard of proof and rules pertaining to legal searches and seizures, can be adjusted to impact both  $\alpha$  and  $\beta$ .

We adapt the approach in Fluet and Demougin (2005) and (2006) to explain how legal procedures affect probabilities of conviction. Specifically, we assume that each person emits a signal x with density g(x) if he commits a crime and with density  $\tilde{g}(x)$  if he does not commit crime, such that  $L(x) \equiv \frac{g(x)}{\tilde{g}(x)}$  is the likelihood ratio with which signal x is produced by individuals who commit the crime versus individuals who refrain from crime. Both density functions have

support [0,1] and satisfy the Monotone Likelihood Ratio Property (MLRP) such that

$$\frac{dL(x)}{dx} < 0 \text{ (MLRP)} \tag{1}$$

which implies that small signals are more inculpatory than large signals. Thus, the legal system sets a threshold signal,  $\bar{x}$ , and convicts individuals who emit signals  $x < \bar{x}$ . This choice implies that individuals who do not commit crime are convicted with probability

$$\alpha(\bar{x}) \equiv \tilde{G}(\bar{x}) \tag{2}$$

where  $\tilde{G}$  is the cumulative distribution function (CDF) associated with  $\tilde{g}$ . Since  $\tilde{G}$  is increasing, the inverse relationship can be noted as follows:

$$\bar{x}(\alpha) = \tilde{G}^{-1}(\alpha) \tag{3}$$

Thus, we may express the probability of convicting people who have committed crimes, as a function of  $\alpha$  instead of a function of  $\bar{x}$ , as follows:

$$\beta(\alpha) = G(\tilde{G}^{-1}(\alpha)) \tag{4}$$

where G is the CDF associated with g. Expressing  $\beta$  as a function of  $\alpha$  instead of  $\bar{x}$  simplifies the analysis by highlighting the trade-off between type-1 and type-2 errors in a very compact manner.

By utilizing MLRP, we can make the following observations regarding the mechanics of this trade-off. First, by differentiating (4), we can note how much a marginal increase in type-1 error reduces type-2 errors, as follows:

$$\beta_{\alpha}(\alpha) = \frac{g(\tilde{G}^{-1}(\alpha))}{\tilde{g}(\tilde{G}^{-1}(\alpha))} = L(\bar{x}(\alpha)) > 0 \text{ for all } \alpha$$
 (5)

Next, we can note that sacrificing wrongful convictions leads to diminishing reductions in type-2 errors since

$$\beta_{\alpha\alpha}(\alpha) = \frac{dL(\bar{x}(\alpha))}{dx} \frac{d\bar{x}(\alpha)}{d\alpha} < 0 \tag{6}$$

due to MLRP and the fact that  $\bar{x}$  is increasing in  $\alpha$ .

Finally, it follows from the definition of  $\beta$  and  $\alpha$  that  $\beta(0) = 0$ ,  $\beta(1) = 1$ , and  $\beta(\alpha) > \alpha$  for all  $\alpha \in (0,1)$ . Figure 2, below, depicts the properties of  $\beta(\alpha)$  under systems with differing levels of accuracy. Thus, figure 2 also illustrates that the relationship between  $\alpha$  and  $\beta$  naturally hinges on the accuracy of the legal system, which is a consideration which we have thus far neglected. Next, we define a general concept of accuracy, and subsequently study the properties it must possess to be consistent with results in prior work.

#### 2.2 Accuracy and Probabilities of Conviction

We conceive of accuracy as the ability of the legal system to distinguish between offenders and non-offenders, which can be accomplished, for instance, by better evidence collection methods. Thus, the impact of increased accuracy is a reduction in the likelihood of type-2 error that must be produced for any targeted probability of type-1 error. Formally, if we let  $a \geq 0$  denote accuracy, we may note this relationship by letting  $\beta$  depend on a, as follows:

$$\beta_a(\alpha, a) > 0$$
 for all  $\alpha \in (0, 1)$ 

A potential relationship between  $\beta$  and a is depicted in figure 2, below, where more accurate systems lead to  $\beta$  functions which circumscribe less accurate systems. The lighter gray curves represent higher accuracy. In particular, the curve with lightest gray represents  $\beta(\alpha, a_2)$ , with  $a_2 > a_1 > a_0$ .

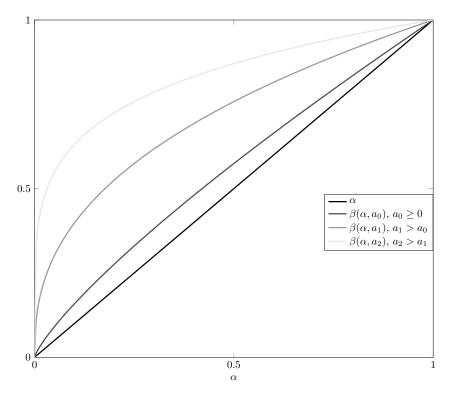


Figure 2: Relationship between  $\alpha$  and  $\beta(\alpha, a)$ 

This broad definition of accuracy corresponds to assuming that a legal system is more accurate if it is characterized by a more informative evidence generating process. However, absent further restrictions, it is not possible to make additional statements about how accuracy impacts the relationship between the two errors.

Most importantly, it is not possible to pin down how exactly accuracy impacts the marginal reductions in type-2 error caused by increases in type-1 error, *i.e.*, the sign of  $\beta_{\alpha a}(\alpha, a)$  cannot be ascertained. Because this relationship is crucial for our analysis, we seek to identify intuitive restrictions which are consistent with assumptions made in the prior literature pertaining to accuracy. This requires explaining the remaining components of our model, and using them to derive results in the prior literature.

#### 3 Preferences and Type-1 Errors

In order to investigate the impact of accuracy on the trade-off between type-1 and type-2 errors, we first explain how these errors affect the incentives of potential offenders. Subsequently, we use these observations to derive optimal type-1 errors, which allows us to identify relationships between accuracy and these policies which are consistent with previous modeling assumptions. Finally, we analyze how the median voter's preferences are impacted by accuracy when it possesses these properties.

#### 3.1 Potential Offenders' Behavior and Deterrence

Potential offenders commit crimes when their expected net-benefits from doing so exceed their expected net-benefits from refraining from crime (Becker, 1968). To deter the commission of crime, the state imposes a punishment whose cost to offenders is normalized to 1. Thus, the potential pay-off from committing crime to a person is  $b - \beta(\alpha, a)$ , where b denotes his criminal gains. These benefits differ from person to person, and we assume they are distributed with density f with support  $[\underline{b}, \overline{b}]$ , and F is the cumulative distribution function associated with f. When a person refrains from crime, he forgoes the option to gain b, and is nevertheless convicted with probability  $\alpha$ . Thus, a person commits crime if his benefit b is such that

$$b > \beta(\alpha, a) - \alpha \equiv b^*(\alpha, a) \tag{7}$$

Here,  $b^*(\alpha, a)$  refers to the deterrence threshold obtained by a given  $\alpha$ , a pair. We assume that  $\underline{b} \leq 0$  and  $\overline{b} > 1$ , such that some people never commit a crime, and others cannot be deterred.<sup>6</sup> This implies that the crime rate can be expressed as  $1 - F(b^*(\alpha, a))$ .

Note that due to the MLRP we have that

$$\beta_{\alpha}(0,a) > 1 > \beta_{\alpha}(1,a) \tag{8}$$

Moreover,  $\beta_{\alpha\alpha}(\alpha, a) < 0$  for all  $\alpha$ , a pairs. Thus, we can implicitly define  $\alpha^b(a)$ , the type-1 error which maximizes deterrence for any given a, as  $\beta_{\alpha}(\alpha^b, a) = 1$ . We note that  $\alpha^b(a)$  plays a critical role in the analysis to follow and that we will refer to it frequently in what follows.

<sup>&</sup>lt;sup>6</sup>The former inequality emerges when the resource costs of committing crime exceed the perceived moral costs of committing crime for some individuals, see, e.g., Benabou and Tirole (2011).

#### 3.2 Welfare and Utility Maximizing Type-1 Errors

Given the responses of potential offenders to different legal procedures, we next investigate both the socially optimal type-1 error as well as the type-1 error that maximizes the median voter's utility. This requires specifying the costs of crime, which come in two varieties. Crimes induce two types of costs. First, the commission of crime causes losses of  $h > \bar{b}$  to others, which we call the external cost of crime. Second, the punishment of an individual generates both a private and a public cost, and their sum represents the social cost of punishment. The public cost per detection equals  $\sigma \in [-1, \bar{\sigma}]$ . When  $\sigma = -1$ , the sanction is fully transferable, which is assumed frequently in the analysis of monetary sanctions.

Punishment costs are financed through lump sum taxes paid by citizens. We assume that both taxes and the harm induced by crime are equally born (or are equally likely to be shared) by each individual in society. The lump sum tax  $\tau(\alpha, a)$  must cover the punishment costs, such that  $\tau(\alpha, a) = \sigma n(\alpha, a)$ , where  $n(\alpha, a)$  is the proportion of convictions (we assume that the size of the population is normalized to 1) which equals

$$n(\alpha, a) = F(b^*(\alpha, a))\alpha + (1 - F(b^*(\alpha, a)))\beta(\alpha, a)$$
(9)

The first and second terms on the right hand-side respectively represent the proportion of individuals wrongfully and correctly convicted.

Next, we express the expected net-benefit of a law abiding citizen (i.e. a person with  $b \leq b^*(\alpha, a)$ ) as

$$u(\alpha, a) = -(1 - F(b^*(\alpha, a)))h - (\alpha + \tau(\alpha, a)) \tag{10}$$

The first term of the expected utility (10) represents the expected gross external harm from crimes (we use the term gross because they exclude private benefits from crimes). The second term represents the (private) expected cost of sanctions, which equal the sum of the law abiding citizen's disutility from being punished in the event of a type-1 error ( $\alpha$ ) and the lump sum tax used to finance the punishment costs ( $\tau(\alpha, a)$ ).

We assume that for all levels of accuracy  $a \ge 0$ , citizens have single-peaked preferences over type-1 errors. Thus, the type-1 error that maximizes the expected net-benefit of a citizen who abides by the law, denoted  $\alpha^u$ , is characterized by  $u_{\alpha}(\alpha^u, a) = 0$  for all  $a \ge 0$ .

We may also express the expected net-benefit of an offender (i.e. a person with  $b > b^*(\alpha, a)$ ) as

$$v(\alpha, a, b) = b - (1 - F(b^*(\alpha, a)))h - (\beta(\alpha, a) + \tau(\alpha, a))$$

$$\tag{11}$$

Like law abiding citizens, offenders are hurt by the external harm from crimes and shoulder the tax burden induced by the public cost of punishment. However, the expected disutility of the sanction is  $\beta(\alpha, a)$  and offenders get an additional benefit b from committing crime.

Social welfare, which consists of the sum of all individuals' utilities can now be expressed as:

$$W(\alpha, a) = \int_{b}^{b^{*}(\alpha, a)} u(\alpha, a) f(b) db + \int_{b^{*}(\alpha, a)}^{\overline{b}} v(\alpha, a, b) f(b) db$$
 (12)

Substituting (10), (11) and (9) in (12), we obtain:

$$W(\alpha, a) = \int_{b^*(\alpha, a)}^{\overline{b}} f(b)(b - h) db - (1 + \sigma)n(\alpha, a)$$
(13)

which implies that social welfare is the difference between the net expected benefit from crime (which is always negative since  $h > \bar{b}$ ) and the social cost of punishment. Note that for purely monetary sanctions ( $\sigma = -1$ ), the latter term equals zero: sanctions are a mere transfer from a benevolent policy maker perspective. We assume that W is single-peaked in  $\alpha$  and for all  $a \geq 0$  such that its unique maximizer, denoted  $\alpha^w(a)$ , is characterized by the first order condition  $W_{\alpha}(\alpha^w, a) = 0$ .

Next, we consider the differences between expected social welfare and the expected utility derived by a law abiding individual. First, the law abiding individual is negatively affected by the expected gross external harm, while the benevolent policy maker cares about the expected net external harm (we use the term net because these include the private benefits of offenders). Second, law abiding citizens are only concerned with their own expected sanction (given by the type-1 error  $\alpha$ ), whereas society is concerned with the expected sanction faced by all citizens (given by the proportion of convicted individuals  $n(\alpha, a)$ ). The consequence of these two differences is that, except for particular values of the model parameters, we may expect the socially optimal type-1 error  $(\alpha^w(a))$ . to be different from the law abiding citizen's preferred type-1 error  $(\alpha^u(a))$ .

To further investigate these differences, we first identify a sufficient condition for both the optimal and law abiding citizens' most preferred enforcement regimes to be interior, *i.e.*  $\alpha \in (0,1)$ . Otherwise, it is possible, for instance, for non-enforcement of laws (*i.e.*  $\alpha = 0$ ) to be both optimal and preferred by law-abiding citizens. The next lemma reveals that these possibilities can be ruled out when the external harm from crime is not small.

Lemma 1 Let

$$\underline{h}(a) \equiv \frac{\beta_{\alpha}(0, a)(\sigma + 1)}{f(0) (\beta_{\alpha}(0, a) - 1)} \tag{14}$$

If h > h(a), then  $\alpha^u(a), \alpha^w(a) \in (0, 1)$ .

**Proof.** First, we find the conditions for  $\alpha^u \in (0,1)$ . We have

$$u_{\alpha}(\alpha, a) = -1 + f(b^*(\alpha, a))b_{\alpha}^*(\alpha, a)h - \sigma n_{\alpha}(\alpha, a)$$
(15)

with

$$n_{\alpha}(\alpha, a) = \beta_{\alpha}(\alpha, a) - F\left(b^{*}(\alpha, a)\right) b_{\alpha}^{*}(\alpha, a) - f\left(b^{*}(\alpha, a)\right) b_{\alpha}^{*}(\alpha, a) b^{*}(\alpha, a) \tag{16}$$

Note that  $\beta(0, a) = 0$ , and thus  $b^*(0, a) = 0$ , which implies  $n_{\alpha}(0, a) = \beta_{\alpha}(0, a)$ . Moreover, we have  $b_{\alpha}^*(0, a) = \beta_{\alpha}(0, a) - 1$ . Substituting and rearranging, we obtain

$$u_{\alpha}(0,a) > 0 \iff h > \frac{\beta_{\alpha}(0,a)\sigma + 1}{f(0)(\beta_{\alpha}(0,a) - 1)} = \underline{\underline{h}}(a)$$
 (17)

With a similar reasoning, we can show that:

$$u_{\alpha}(1,a) < 0 \Leftrightarrow h > \frac{\beta_{\alpha}(1,a)\sigma + 1}{f(0)(\beta_{\alpha}(1,a) - 1)}$$

$$\tag{18}$$

Which is always satisfied since  $\beta_{\alpha}(1, a) < 1$  from MLRP. Thus the right-hand side of the last condition in (18) is always negative.

Second, we find the conditions for  $\alpha^w \in (0,1)$ . We have

$$W_{\alpha}(\alpha, a) = -f(b^*(\alpha, a))b_{\alpha}^*(\alpha, a)(b^*(\alpha, a) - h) - (1 + \sigma)n_{\alpha}(\alpha, a) \tag{19}$$

After substituting and rearranging, we obtain

$$W_{\alpha}(0,a) > 0 \iff h > \frac{\beta_{\alpha}(0,a)(\sigma+1)}{f(0)(\beta_{\alpha}(0,a)-1)} = \underline{h}(a)$$
 (20)

With a similar reasoning, we can show that

$$W_{\alpha}(1,a) < 0 \iff h > \frac{\beta_{\alpha}(1,a)(\sigma+1)}{f(0)(\beta_{\alpha}(1,a)-1)}$$
 (21)

This last condition is always satisfied. Note that  $\underline{h}(a) \geq \underline{\underline{h}}(a)$  for all  $a \geq 0$ . Thus, if  $h > \underline{h}(a)$ , then  $\alpha^u(a), \alpha^w(a) \in (0,1)$ .

The rationale behind lemma 1 follows from the fact that as the external harm from crime increases, its minimization becomes a more important concern (both for citizens as well as the social objective) relative to the minimization of punishment costs. The former objective is achieved by the type-1 error which maximizes deterrence (i.e.  $\alpha^b(a)$ ), which is interior, whereas the latter objective is achieved through corner solutions, i.e.  $\alpha \in \{0,1\}$ . Thus, for sufficiently large external harms, the relative importance of the deterrence objective pulls both the optimal as well as citizens' most preferred type-1 error closer to  $\alpha^b(a)$ , making them both interior.

Since we are interested in cases where some enforcement takes place, unless otherwise specified, we consider values of h and a such that  $h > \underline{h}(a)$ . With this assumption in place, we proceed by investigating when and how optimal legal procedures differ from those preferred by law abiding citizens. Proposition 1, below, summarizes our findings.

**Proposition 1** (i) If private punishment costs are completely transferable, the deterrence maximizing type-1 error is both optimal and most preffered by law abiding citizens, i.e. if  $\sigma = -1$ , then  $\alpha^w(a) = \alpha^u(a) = \alpha^b(a)$ . (ii) If punishment costs are not completely transferable, then the optimal type-1 error is smaller than that preferred by law-abiding citizens, which itself is smaller than the deterrence maximizing type-1 error, i.e. if  $\sigma > -1$ , then  $\alpha^w(a) < \alpha^u(a) < \alpha^b(a)$ .

**Proof.** (i) If  $\sigma = -1$ , the FOCs characterizing  $\alpha^u(a)$  and  $\alpha^w(a)$  can be expressed as

$$b_{\alpha}^{*}(\alpha^{u}(a), a) \left[ f\left(b^{*}(\alpha^{u}(a), a)\right) \left(h - b^{*}(\alpha^{u}(a), a)\right) + \left(1 - F\left(b^{*}(\alpha^{u}(a), a)\right)\right) \right] = 0$$
(22)

and

$$f(b^*(\alpha^w(a), a))b_{\alpha}^*(\alpha^w(a), a)(h - b^*(\alpha^w(a), a)) = 0,$$
(23)

respectively. Both conditions are satisfied only if  $b_{\alpha}^* = 0$ , which implies that  $\alpha^u(a) = \alpha^w(a) = \alpha^b(a)$ .

(ii) First, we show (through contradiction) that  $\alpha^u(a,\sigma) < \alpha^b(a)$  where we express  $\alpha^u = \alpha^u(a,\sigma)$  to note the dependency of  $\alpha^u$  to the public cost of punishment. The FOC characterizing  $\alpha^u$  is

$$u_{\alpha}(\alpha, a, \sigma) = -1 + f(b^*(\alpha, a))b_{\alpha}^*h - \sigma n_{\alpha}(\alpha, a)$$
(24)

Thus,

$$\alpha_{\sigma}^{u}(a,\sigma) = -\frac{u_{\alpha\sigma}(\alpha^{u}, a, \sigma)}{u_{\alpha\alpha}(\alpha^{u}, a, \sigma)} \le 0 \text{ iff } n_{\alpha} \ge 0$$
 (25)

since  $u_{\alpha\alpha}(\alpha^u, a, \sigma) < 0$ .

Next, suppose that there exists  $\sigma' > -1$  such that  $\alpha^u(a, \sigma') \ge \alpha^b(a)$ . This implies that  $b_{\alpha}(\alpha^u(a, \sigma')) = \beta_{\alpha}(\alpha^u(a, \sigma')) - 1 \le 0 = \beta_{\alpha}(\alpha^b(a)) - 1 = b_{\alpha}(\alpha^b(a))$  since  $\beta$  is concave in  $\alpha$  due to the MLRP. Thus, per (16), it follows that  $n_{\alpha} > 0$ , which implies via (25), that  $\alpha^u_{\sigma}(a, \sigma') < 0$ . This, in turn implies that  $\alpha^u(a, \sigma) > \alpha^u(a, \sigma') > \alpha^b(a)$  for all  $\sigma \in [-1, \sigma')$ , which is a contradiction with the fact that  $\alpha^u(a, -1) = \alpha^b(a)$ . Thus,  $\alpha^u(a, \sigma') < \alpha^b(a)$  for all  $\sigma > -1$ .

Second, we compare  $\alpha^u(a)$  to  $\alpha^w(a)$  (where we drop  $\sigma$  as an argument, since it no longer plays a role in the proof). We have

$$W_{\alpha}(\alpha, a) = u_{\alpha}(\alpha, a) - (1 - F(b^*(\alpha, a)))b_{\alpha}^*(\alpha, a) \tag{26}$$

Note that for all  $\alpha < \alpha^b(a)$ , we have that  $b_{\alpha}^*(\alpha, a) = \beta_{\alpha}(\alpha, a) - 1 > 0 = \beta_{\alpha}(\alpha^b(a), a) - 1 = b_{\alpha}(\alpha^b(a), a)$  since  $\beta$  is concave in  $\alpha$  due to the MLRP. Thus,

$$W_{\alpha}(\alpha, a) < u_{\alpha}(\alpha, a) \ \forall \alpha \in [0, \alpha^{b}(a))$$
 (27)

This implies that  $W_{\alpha}(\alpha^{u}(a), a) < u_{\alpha}(\alpha^{u}(a), a) = 0$  since  $\alpha^{u}(a) < \alpha^{b}(a)$  as shown in the first step. This, in turn, implies that  $\alpha^{w}(a) < \alpha^{u}(a)$  since W is single peaked.

First, we explain the intuition behind the result that  $\alpha^w(a) = \alpha^b(a)$  when  $\sigma = -1$ . The sanction is a pure transfer, and therefore there is no direct social cost associated with punishment. Thus, since the external harm is higher than the maximal benefit from crime, i.e.,  $h > \bar{b}$ , the objective of the benevolent policy maker is to maximize deterrence. As a result, the socially optimal type-1 error  $(\alpha^w(a))$  equals the type-1 error that maximizes deterrence  $(\alpha^b(a))$ .

Second, we similarly explain the rationale behind the result that  $\alpha^u(a) = \alpha^b(a)$  when  $\sigma = -1$ . Type-1 errors affect the crime rate as well as the percapita tax revenue received net of expected punishment costs for law abiding

citizens. A reduction in the crime rate always benefits law abiding citizens, since this reduces the criminal harms expected to be inflicted on them and it also increases the tax revenue receivable by law abiding citizens. Indeed, the crime rate is minimized by  $\alpha^b(a)$ , which maximizes the expected benefits related to changes in the crime rate for law abiding citizens. Moreover, the percapita tax revenue net of expected punishment costs for law abiding citizens is the difference between  $n(\alpha)$  and  $\alpha$ , which equals the crime rate multiplied by the discriminatory power of the legal standards employed (i.e.  $\beta(\alpha, a) - \alpha$ ). This gap, too, is maximized by  $\alpha^b(a)$ . Thus,  $\alpha^b(a)$  maximizes both gains from changes in the crime rate as well as the tax revenue net of expected punishment costs given the crime rate, which makes  $\alpha^b(a)$  the most preferred type-1 error of law abiding citizens.

Third, we explain why  $\alpha^w(a) < \alpha^u(a)$  when  $\sigma > -1$ , and to do so, we consider the impact of lowering  $\alpha$  slightly below  $\alpha^b(a)$ . This leads to a change in the criminal harms inflicted as well as a change in the expected disutility from punishment. To explain the impact of the former effect, recall that part of the benevolent policy maker's objective is to minimize the expected net external harm while, in comparison, the law abiding citizen is concerned only with the expected gross external harm. Following a small decrease in the type-1 error below  $\alpha^b(a)$ , deterrence is decreased, which naturally increases the total gross external harm by more than the net external harm. Thus, law abiding citizens internalize a greater impact related to changes in criminal behavior than society as a whole. The second effect occurs through differences in the impact on the expected disutility from punishment. The law-abiding citizen only internalizes her own expected punishment (of magnitude  $\alpha$ ) whereas a benevolent policy maker is concerned about the expected disutility of all citizens (or equivalently, the average expected disutility of sanctions). Following a small decrease in the type-1 error below  $\alpha^b(a)$ , both disutilities are decreased (since convictions occur less frequently). However, the reduction in the conviction probability is greater for an offender than for a law abiding citizen. As a consequence, the positive impact of the small decrease in  $\alpha$  is larger on the average person's expected disutility than on the law abiding citizen's expected disutility. Thus, the marginal social gains from a reduction in  $\alpha$  below  $\alpha^b$  is greater than the marginal gains for a law abiding citizen. In sum, reducing  $\alpha$  below  $\alpha^b$  generates a lower marginal cost and a larger marginal gain for society than for the law abiding citizen, which causes the socially optimal type-1 error  $(\alpha^w(a))$  to be lower than the law abiding citizen's preferred type-1 error  $(\alpha^u(a))$ .

Finally, we explain why  $\alpha^w(a) < \alpha^b(a)$  and  $\alpha^u(a) < \alpha^b(a)$  when  $\sigma > -1$ . The expected cost of punishment is positive, since the disutility from punishment is greater than any tax revenue that may be obtained. Lowering  $\alpha$  slightly below  $\alpha^b(a)$  decreases these expected costs of punishment by reducing the probability of conviction by sacrificing a negligible degree of deterrence. Thus, both the socially optimal type-1 error  $(\alpha^w(a))$  and the law abiding citizen's preferred type-1 error  $(\alpha^u(a))$  are lower than the type-1 error which maximizes deterrence  $(\alpha^b(a))$ .

#### 3.3 Impact of Accuracy on Optimal Errors

Here, we study the impact of accuracy on optimal type-1 errors and note two important insights that are revealed from the analysis. First, we point out that absent restrictive assumptions one cannot generally ascertain the direction towards which optimal type-1 and type-2 errors move in response to increases in accuracy. This is important, because the narrow literature on accuracy, most notably Kaplow and Shavell (1994), make the assumption that increased accuracy reduces both type-1 and type-2 errors. However, as we demonstrate below, for optimal type-1 and type-2 errors to be monotonically decreasing in accuracy, the trade-off between type-1 and type-2 errors must be affected in a particular direction through an increase in accuracy around the optimal error pair. Specifically, the reduction in one type of error that can be achieved through the increase in the other type of error must be decreasing in accuracy, i.e.  $\beta_{\alpha a}(\alpha^w(a), a) < 0$ .

To demonstrate these insights, we first note that the manner in which accuracy affects the first order condition characterizing the optimal type-1 error, and therefore the sign of  $W_{\alpha a}(\alpha^w(a), a)$  cannot be ascertained. We illustrate this fact by focusing on the simplest and most frequently analyzed case in the literature where the sanction is completely transferable, *i.e.*  $\sigma = -1$ . In this case, the first order condition simplifies to

$$W_{\alpha}(\alpha, a) = f(b^*(\alpha, a))b_{\alpha}^*(\alpha, a) \left(h - b^*(\alpha, a)\right) \tag{28}$$

and, thus, the socially optimal type-1 error is characterized by

$$\beta_{\alpha}(\alpha^w, a) - 1 = 0 \tag{29}$$

This leads to two important observations. First, this condition is the same as the one used to characterize  $\alpha^b(a)$ . Indeed, as shown in proposition 1,  $\alpha^w(a) = \alpha^b(a)$  if  $\sigma = -1$ . Second, as explained in Demougin and Fluet (2005) and Mungan (2020a),  $\alpha^b(a)$  can be interpreted as the *preponderance of the evidence* standard of proof. Using the implicit function theorem reveals that

$$\alpha_a^b(a) = -\frac{\beta_{\alpha a}(\alpha^b(a), a)}{\beta_{\alpha \alpha}(\alpha^b(a), a)} \tag{30}$$

From MLRP, the denominator of (30) is negative and thus

$$\alpha_a^b(a) < 0 \iff \beta_{\alpha a}(\alpha^b(a), a) < 0$$
 (31)

A similar result follows for type-2 errors, *i.e.*  $1 - \beta(\alpha^b(a), a)$ :

$$\frac{\mathrm{d}\left(1 - \beta(\alpha^b(a), a)\right)}{\mathrm{d}a} < 0 \iff \frac{\beta_{\alpha a}(\alpha^b(a), a)}{\beta_a(\alpha^b(a), a)} > \beta_{\alpha \alpha}(\alpha^b(a), a) \tag{32}$$

Thus, the effect of increasing accuracy on the type-1 and type-2 errors that maximizes deterrence (and expected welfare when  $\sigma = -1$ ) crucially depends on how

accuracy affects the evidence generating and interpretation process. Therefore, caution is required when arguing that errors decrease with accuracy, since this may not be true. Note however that both types of errors cannot simultaneously increase following an increase in accuracy. We discuss the remaining three cases with graphical illustrations and provide examples of functional forms that possess the properties depicted in footnotes.

Figure 3: Variations of type-1 and type-2 errors when a increases

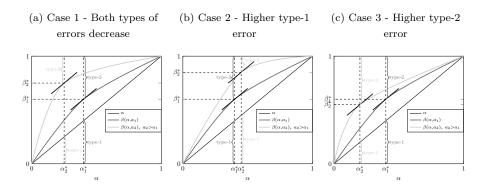


Figure 3(a) illustrates the case in which both types of error simultaneously decrease with a (i.e. (31) and (32) are both satisfied). Following an increase in the level of accuracy (from  $a_1$  to  $a_2$ , with  $a_2 > a_1$ ), for a given interior type-1 error (i.e.  $\alpha \in (0,1)$ ) the policy maker is better able to distinguish the innocent from the guilty (i.e.  $\beta(\alpha,a_2) > \beta(\alpha,a_1)$ ). In figure 3(a), this implies that the light gray curve is above the dark gray curve. The first order condition (29) implies that graphically the optimal type-1 error is obtained when a line with slope 1 is tangent to the curve depicting  $\beta$ . In the case depicted in figure 3(a), an increase in accuracy from  $a_1$  to  $a_2$  causes this tangency condition to be satisfied at a point that lies to the northwest of where it used to be satisfied. Thus, both type-1 and type-2 errors are lowered.

Optimal legal errors move in opposite directions in response to a change in accuracy in the cases depicted in figures 3(b) and (c): optimal type-1 errors are increasing while optimal type-2 errors are decreasing in accuracy in figure 3(b), and we have the opposite relationship in figure 3(c). In both cases, however, a simple implication of the envelope theorem, applied to the first order condition

$$\beta_1(\alpha, a) = \frac{2^a}{2^a - 1} \left( 1 - \frac{1}{(1 + \alpha)^a} \right)$$

<sup>&</sup>lt;sup>7</sup>As previously noted, Kaplow and Shavell (1994) assume that increased accuracy always reduces both type-1 and type-2 errors. As we explain in section 2.2., our definition of accuracy is more general and only suggests that more accuracy allows one to better separate an innocent person from a guilty person given any desired type-1 error (*i.e.* that  $\beta(\alpha, a) - \alpha$  is increasing with a for all  $\alpha \in (0, 1)$ ).

<sup>&</sup>lt;sup>8</sup>A functional form that produces this result is  $\beta(\alpha, a) = \gamma \beta_1(\alpha, a) + (1 - \gamma)\beta_2(\alpha)$  where

(29), is that deterrence is increasing with the level of accuracy. This means that the increase in one type of error is more than compensated by the decrease in the other type of error, resulting globally in a lower sum of the probabilities of error, and thus greater deterrence (*i.e.* a lower crime rate).

Our discussion reveals that, in general, one cannot unambiguously ascertain the direction towards which optimal type-1 and type-2 errors move in response to increases in accuracy. Thus, we note a technical necessary condition for the intuitive relationship assumed in the literature to hold, as follows.

**Proposition 2** Suppose the optimal type-1 error is monotonically decreasing in accuracy (i.e.  $\alpha_a^w(a) < 0$ ) for all  $\sigma \in [-1, \overline{\sigma}]$ , then  $\beta_{\alpha a}(\alpha^b(a), a) < 0$ .

**Proof.** As noted above, if  $\sigma = -1$  then  $\alpha^w(a) = \alpha^b(a)$ . Thus, (31) is a necessary condition for  $\alpha^w(a)$  to be monotonically decreasing in a.

Proposition 2 reveals that for optimal type-1 errors to be decreasing in response to accuracy, the trade-off between type-1 and type-2 errors must display a type of diminishing returns from accuracy. Whether this property holds is determined entirely by how accuracy impacts the evidentiary process, and requires a restriction beyond the simple requirement that more accuracy causes the evidentiary process to be more informative.

Unless otherwise specified, we will assume in the remaining of our analysis that the optimal type-1 error is monotonically decreasing in accuracy, and thus that  $\beta_{\alpha a}(\alpha^b(a), a) < 0$ . Moreover, we also make the following technical assumption on the same cross-derivative to ease the derivation of results.

**Assumption 1.**  $\beta_{\alpha a}(\alpha^b(a), a)$  is bounded, *i.e.* there exists c > 0 such that  $|\beta_{\alpha a}(\alpha^b(a), a)| \le c$  for all  $a \ge 0$ .

# 3.4 Impact of Accuracy on the Median Voter's Most Preferred type-1 Error

Next, we investigate how the median voter's most preferred error is related to accuracy when there are diminishing returns from accuracy. In our analysis, we focus on cases where less than half of the population commit offenses under the relevant range of policies.<sup>9</sup> There are two simple justifications for this assumption in addition to the fact that it allows us to abbreviate derivation of results. First, empirically, it appears realistic to assume that a small proportion of individuals derive significant private benefits from crime.<sup>10</sup> Second,

and:

$$\beta_2(\alpha) = 1 - (1 - \alpha)^{\frac{1}{1 - \epsilon}}$$

We adapt  $\beta_1(\alpha,a)$  from Lundberg and Mungan (2020) where the functional form was used to illustrate possibilities in a different context. More specifically, examples consistent with cases 1-3 depicted in figures 3a-c are obtained by letting  $a_1=1 < a_2=2$  and  $\gamma=1$  (for case 1),  $\gamma=0.25$  and  $\epsilon=0.9$  (for case 2), and  $\gamma=0.25$  and  $\epsilon=0.1$  (for case 3).

<sup>&</sup>lt;sup>9</sup>When  $F(0) \ge 0.5$  this assumption always holds, but it can hold in many other circumstances where F(0) < 0.5 as we note in the remainder of our discussion.

<sup>&</sup>lt;sup>10</sup>An alternative way of incorporating this idea is to explicitly model criminal tendencies, as opposed to the criminal benefits of potential offenders, and assume that the average criminal

earlier contributions (Langlais and Obidzinski 2017, Obidzinski 2019) where the majority is endogenously determined, have shown that as long as there are sufficiently large harms from crime, the majority consists of law abiding citizens in equilibrium.

With this assumption in place, our previous findings in proposition 1 imply that an electoral process<sup>11</sup> which implements the median voter's most preferred policy will tend to generate greater than optimal type-1 errors. Next, we summarize how the median voter's most preferred type-1 error is affected by increased accuracy.

**Proposition 3** There exists a threshold level of harm,  $\bar{h}$ , such that  $h > \bar{h}$ implies that  $\alpha_a^u(a) < 0$ .

**Proof.** We can rewrite the first order condition for  $\alpha^u(a)$  as

$$k(\alpha, a) - \frac{1}{h}l(\alpha, a) = 0 \tag{33}$$

where  $k(\alpha, a) = f(b^*(\alpha, a))b_{\alpha}^*(\alpha, a)$  and  $l(\alpha, a) = 1 + \sigma n_{\alpha}(\alpha, a)$ . It follows that  $\lim_{h\to +\infty} [k(\alpha,a)-\frac{1}{h}l(\alpha,a)]=k(\alpha,a)$ , and, thus  $\lim_{h\to +\infty}\alpha^u(a)=\alpha^b(a)$ , since  $k(\alpha,a)=0 \Leftrightarrow b_{\alpha}^*=0$ . Therefore,

$$\lim_{h \to +\infty} b_{\alpha}^*(\alpha^u(a), a) = b_{\alpha}^*(\alpha^b(a), a) = 0$$
(34)

We note that

$$\alpha_a^u(a) = -\frac{k_a(\alpha^u(a), a) - \frac{1}{\hbar}l_a(\alpha^u(a), a)}{\frac{\partial [k(\alpha, a) - \frac{1}{\hbar}l(\alpha, a)]}{\partial \alpha}}$$
(35)

with  $\frac{\partial [k(\alpha,a) - \frac{1}{h}l(\alpha,a)]}{\partial \alpha} < 0$ , since  $\alpha^u(a)$  is a proper maximum. Thus,

$$\operatorname{sign}\left[\lim_{h\to+\infty}\alpha_a^u(a)\right] = \operatorname{sign}\left[\lim_{h\to+\infty}\left[k_a(\alpha^u(a),a) - \frac{1}{h}l_a(\alpha^u(a),a)\right]\right] (36)$$

$$= \operatorname{sign}\left[\lim_{h \to +\infty} k_a(\alpha^b(a), a)\right]$$
 (37)

$$= \operatorname{sign}\left[\beta_{\alpha a}(\alpha^b(a), a) f(b^*(\alpha^b(a), a))\right] < 0$$
 (38)

where the second equality follows from the facts that  $l_a(\alpha^b(a), a)$  is bounded (per assumption 1) and is independent of h, the third equality follows from (34) and the inequality in the last line holds since  $\beta_{\alpha a}(\alpha^b(a), a) < 0$  (as noted in proposition 2). Thus, if h is large enough, then we have  $\alpha_a^u(a) < 0$ .

tendency is greater than the median voter's criminal tendency. This setting generates similar

implications, see Mungan (2017).  $$^{11}\rm{Previous}$  works have analyzed the choice of monetary and non monetary sanctions, detection, and accuracy under Downsian electoral competition (Langlais and Obidzinski 2017, Obidzinski 2019) or through the median voter theorem (Mungan 2017).

Proposition 3 states that an increase in accuracy reduces the median voter's preferred type-1 error, as long as the harm induced by the crime is high enough. When the external harm from crime (h) increases, the citizen's concern for the expected external harm becomes more important relative to her concern for the expected cost of sanctions. As a result, the citizen's objective tends toward the maximization of deterrence. As we previously noted, when there are diminishing returns from increased accuracy, in the form of reduced type-2 error from increasing type-1 error, it follows that the deterrence maximizing type-1 error is lowered with greater accuracy. Thus, for large enough harms, the median voter's most preferred type-1 error is also decreased, since it mimics the properties of the deterrence maximizing type-1 error.

The dynamics we outline above, of course, need not hold for small harm crimes. This is because when harms are small and the enforcement mechanism is inaccurate, the median voter need not be as concerned about deterrence as she is with other objectives (*i.e.*, tax consequences and limiting her expected wrongful conviction costs). This may cause her to prefer very low type-1 errors. However, for type-1 errors which are much below the deterrence maximizing type-1 error, increased accuracy enhances the effectiveness of type-1 errors in eliminating type-2 errors, i.e.  $\beta_{\alpha a}(\alpha, a) > 0$  for sufficiently small  $\alpha$ . Thus, when harms are small, the median voter's most preferred type-1 error can increase with more accuracy. We formalize this result by constructing a simple example.

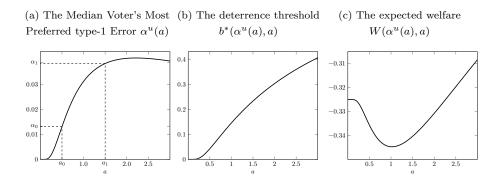
**Example 1** For 
$$b \in [0, \overline{b}]$$
, let  $f(b) = 0.5/\overline{b}$ , such that  $F(b) = 0.5(1 + \frac{b}{\overline{b}})$ . Moreover, let  $h = 1.2$ ,  $\sigma = 0$ ,  $\overline{b} = 1.1$  and  $\beta(\alpha, a) = \alpha^{\frac{1}{1+a}}$ .

It is easy to verify that the specified  $\beta$  function satisfies all of the properties we have discussed in previous sections. Specifically,  $\beta(0, a) = 0$ ,  $\beta(1, a) = 1$ ,  $\beta(\alpha, a) > 0 > \beta_{\alpha a}(\alpha, a)$  for all  $\alpha \in (0, 1)$ .

In Figure 4(a), we plot the median voter's most preferred type-1 error  $\alpha^u(a)$  obtained in our example as a function of accuracy. Since we take h as given when we vary accuracy, it follows that for small enough a, the harms from crime are insufficient to warrant enforcement from the median voter's perspective, thus  $\alpha^u(a) = 0$  for small a. However, when a is not small, the median voter prefers some enforcement, and her most preferred type-1 error is increasing in accuracy. Some numerical values that emerge from our example are marked with dashed lines to illustrate this fact. Specifically,  $\alpha_0 = \alpha^u(0.5) \approx 0.01303 < \alpha_1 = \alpha^u(1.5) \approx 0.03828$ . This example is used to formalize the following result.

**Proposition 4** For small h, an increase in accuracy may lead to an increase in the median voter's most preferred type-1 error (i.e. it is possible that  $\alpha_a^u(a) > 0$ ).

Figure 4: Numerical example with  $a \in (0,3)$ 



Next, we highlight an additional and counter-intuitive result that is revealed through Example 1 (and illustrated via figures 4(b) and (c)).

**Proposition 5** An increase in accuracy may cause a reduction in welfare despite enhancing deterrence when the median voter's most preferred policy is implemented.

Figures 4(b) and (c) graphically illustrate how an increase in accuracy may cause a reduction in social welfare despite enhancing deterrence. The figures plot welfare and the deterrence threshold obtained when the median voter's most preferred policy is implemented. They illustrate that the joint increases in the level of accuracy and type-1 error (caused by greater accuracy) have a positive effect on deterrence, but a negative effect on expected welfare. The intuition behind the positive effect on deterrence is that, first, as shown in proposition 1,  $\alpha^u(a) < \alpha^b(a)$  if  $\sigma > -1$ . Thus, a small increase in the type-1 error above  $\alpha^u(a)$ increases deterrence (as it does whenever  $\alpha < \alpha^b(a)$ ). Second, ceteris paribus, an increase in the level of accuracy always has a positive impact on deterrence. The intuition behind the negative effect on expected welfare is that, as shown in proposition 1,  $\alpha^u(a) > \alpha^w(a)$  if  $\sigma = 0$ . Thus, a small increase in the type-1 error above  $\alpha^{u}(a)$  may increase the gap between the median voter's most preferred type-1 error and the socially optimal one, causing a decrease in expected welfare as a result. Thus, quite counter-intuitively, an increase in accuracy can lead to loss of welfare when the median voter's most preferred policy is implemented due to electoral pressures.

#### 4 Conclusion

The evolution of many criminal law processes towards reducing type-1 errors is an interesting phenomenon. Here we have questioned whether this type of evolution is consistent with changes in the popular demand for legal institutions as a function of increased accuracy in the determination of guilt. Our findings

suggest that for serious violations which generate large social harms, the median voter's preferred type-1 error is decreasing with accuracy, which is consistent with historical trends. On the other hand, the same conclusion need not hold for lesser wrongdoings, e.g. civil infractions. Our analysis also revealed that legal procedures that emerge under electoral pressures generate above optimal type-1 errors, i.e. wrongfully convict or impose liability more frequently than is optimal. Moreover, contrary to intuition, increases in accuracy can be welfare reducing. Overall, our analysis provides a new perspective through which one can study and interpret the evolution of legal institutions, and it draws attention to specific ways in which electoral pressures can contribute to the emergence of inefficient legal procedures.

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